M II - 2014

Určete definiční obor fce $f(x)$, spočtěte tečnu v bodě a.

(1) $f(x) = \operatorname{arctg}\left(x - \dfrac{2}{x} + 1\right), \qquad a = 1.$

(2) $f(x) = \ln\left(x^2 + 3x + 1\right), \qquad a = -3.$

(3) $f(x) = \sqrt{\mathrm{e}^{1-x^2}}, \qquad a = 1.$

(4) $f(x) = \mathrm{e}^{\frac{3-x}{x-2}}, \qquad a = 3.$

(5) $f(x) = (1 - \ln(2-x))^5, \qquad a = 1.$

(6) $f(x) = \mathrm{e}^{\sin\left(1-x^2\right)}, \qquad a = -1.$

(7) $f(x) = \sqrt{\cos(2x)}, \qquad a = 0.$

(8) $f(x) = \mathrm{e}^{x^2}\operatorname{arctg}(x), \qquad a = 0.$

(9) $f(x) = \operatorname{arctg}\left(\mathrm{e}^{1-x}\right), \qquad a = 1.$

(10) $f(x) = \dfrac{\sin(x) + 3}{1 - \sin(x)}, \qquad a = 0.$

(11) $f(x) = \cos\left(\dfrac{x}{1-x^2}\right), \qquad a = 0.$

(12) $f(x) = \mathrm{e}^{\operatorname{arctg}(1-x)}, \qquad a = 1.$

(13) $f(x) = \dfrac{\mathrm{e}^{2x}}{2 - \mathrm{e}^{2x}}, \qquad a = 0.$

(14) $f(x) = \dfrac{1}{\sqrt{1 - x - 2x^2}}, \qquad a = 0.$

(15) $f(x) = \sin\left(1 - x - 2x^2\right), \qquad a = -1.$

(16) $f(x) = \operatorname{arctg}\left(\mathrm{e}^{x^2}\right), \qquad a = 0.$

(17) $f(x) = \mathrm{e}^x \sin(2x), \qquad a = 0.$

(18) $f(x) = \ln\left(x + \dfrac{2}{x} - 2\right), \qquad a = 1.$

(19) $f(x) = \mathrm{e}^{\sqrt{1-x^2}}, \qquad a = 0.$

(20) $f(x) = \sin\left(\dfrac{x}{x^2+1}\right), \qquad a = 0.$

(21) $f(x) = \sqrt{1 - x - 2x^2}, \qquad a = 0.$

(22) $f(x) = e^{-x}\cos(3x), \qquad a = 0.$

(23) $f(x) = \mathrm{e}^{\frac{x}{x^2+1}}, \qquad a = 0.$

(24) $f(x) = \operatorname{tg}\left(x - \dfrac{2}{x} + 1\right), \qquad a = 1.$

(25) $f(x) = \dfrac{1}{\sqrt{\sin(x)}}, \qquad a = \dfrac{\pi}{2}.$

(26) $f(x) = \left(3 - \mathrm{e}^{2-x}\right)^3, \qquad a = 2.$

(27) $f(x) = \ln\left(x^2 - \sqrt{x} + 1\right), \qquad a = 1.$

(28) $f(x) = \sin\left(\dfrac{1}{x} - x\right), \qquad a = 1.$

(29) $f(x) = \dfrac{1 + \cos(x)}{2 - \cos(x)}, \qquad a = 0.$

(30) $f(x) = (\sin(2-x) + 1)^7, \qquad a = 2.$

Určete definiční obor fce $f(x)$, spočtěte tečnu v bodě a.

(1) $\mathcal{D}(f): x \neq 0, \quad f'(x) = \dfrac{\frac{2}{x^2}+1}{\left(x-\frac{2}{x}+1\right)^2+1}, \quad t: y = 3x-3.$

(2) $\mathcal{D}(f): x < \dfrac{1}{2}\left(-3-\sqrt{5}\right) \vee x > \dfrac{1}{2}\left(\sqrt{5}-3\right), \quad f'(x) = \dfrac{2x+3}{x^2+3x+1}, \quad t: y = -3x-9.$

(3) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -x\sqrt{\mathrm{e}^{1-x^2}}, \quad t: y = 2-x.$

(4) $\mathcal{D}(f): \mathbb{R}\setminus\{2\}, \quad f'(x) = -\dfrac{\mathrm{e}^{\frac{3-x}{x-2}}}{(x-2)^2}, \quad t: y = 4-x.$

(5) $\mathcal{D}(f): (-\infty, 2), \quad f'(x) = -\dfrac{5(\ln(2-x)-1)^4}{x-2}, \quad t: y = 5x-4.$

(6) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -2x\mathrm{e}^{\sin\left(1-x^2\right)}\cos\left(1-x^2\right), \quad t: y = 2x+3.$

(7) $\mathcal{D}(f): \left\langle -\dfrac{\pi}{4}, \dfrac{\pi}{4}\right\rangle$ **perioda** π, $\quad f'(x) = -\dfrac{\sin(2x)}{\sqrt{\cos(2x)}}, \quad t: y = 1.$

(8) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = \mathrm{e}^{x^2}\left(\dfrac{1}{x^2+1}+2x\,\mathrm{arctg}(x)\right), \quad t: y = x.$

(9) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{\mathrm{e}^{1-x}}{(\mathrm{e}^{1-x})^2+1}, \quad t: y = -\dfrac{x}{2}+\dfrac{\pi}{4}+\dfrac{1}{2}.$

(10) $\mathcal{D}(f): \mathbb{R}\setminus\left\{\dfrac{\pi}{2}+2\,k\pi\right\}, \quad f'(x) = \dfrac{4\cos(x)}{(\sin(x)-1)^2}, \quad t: y = 4x+3.$

(11) $\mathcal{D}(f): \mathbb{R}\setminus\{\pm 1\}, \quad f'(x) = -\dfrac{\left(x^2+1\right)\sin\left(\frac{x}{1-x^2}\right)}{\left(x^2-1\right)^2}, \quad t: y = 1.$

(12) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{\mathrm{e}^{\mathrm{arctg}(1-x)}}{(1-x)^2+1}, \quad t: y = 2-x.$

(13) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = \dfrac{4\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-2\right)^2}, \quad t: y = 4x+1.$

(14) $\mathcal{D}(f): -1 < x < \dfrac{1}{2}, \quad f'(x) = \dfrac{4x+1}{2\left(-2x^2-x+1\right)^{3/2}}, \quad t: y = \dfrac{x}{2} + 1.$

(15) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -(4x+1)\cos\left(-2x^2-x+1\right), \quad t: y = 3x+3.$

(16) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = \dfrac{2e^{x^2}x}{e^{2x^2}+1}, \quad t: y = \dfrac{\pi}{4}.$

(17) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = e^x(\sin(2x) + 2\cos(2x)), \quad t: y = 2x.$

(18) $\mathcal{D}(f): x > 0, \quad f'(x) = \dfrac{x^2-2}{x\left(x^2-2x+2\right)}, \quad t: y = 1-x.$

(19) $\mathcal{D}(f): \langle -1, 1\rangle, \quad f'(x) = -\dfrac{xe^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}, \quad t: y = e.$

(20) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{\left(x^2-1\right)\cos\left(\frac{x}{x^2+1}\right)}{\left(x^2+1\right)^2}, \quad t: y = x.$

(21) $\mathcal{D}(f): -1 \leq x \leq \dfrac{1}{2}, \quad f'(x) = \dfrac{-4x-1}{2\sqrt{-2x^2-x+1}}, \quad t: y = 1 - \dfrac{x}{2}.$

(22) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -e^{-x}(3\sin(3x) + \cos(3x)), \quad t: y = 1-x.$

(23) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{e^{\frac{x}{x^2+1}}\left(x^2-1\right)}{\left(x^2+1\right)^2}, \quad t: y = x+1.$

(24) $\mathcal{D}(f): x \neq 0,$ **app.** $, \quad f'(x) = \dfrac{\left(\frac{2}{x^2}+1\right)}{\cos^2\left(x - \frac{2}{x} + 1\right)}, \quad t: y = 3x-3.$

(25) $\mathcal{D}(f): (0, \pi)$ **perioda** $2\pi, \quad f'(x) = -\dfrac{\cos(x)}{2\sqrt{\sin^3(x)}}, \quad t: y = 1.$

(26) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = 3\left(3 - e^{2-x}\right)^2 e^{2-x}, \quad t: y = 12x - 16.$

(27) $\mathcal{D}(f): x \geq 0, \quad f'(x) = -\dfrac{\frac{1}{\sqrt{x}} - 4x}{2x^2 - 2\sqrt{x} + 2}, \quad t: y = \dfrac{3x}{2} - \dfrac{3}{2}.$

(28) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{\left(x^2+1\right)\cos\left(\frac{1}{x} - x\right)}{x^2}, \quad t: y = -2x+2.$

(29) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -\dfrac{3\sin(x)}{(\cos(x)-2)^2}, \quad t: y = 2.$

(30) $\mathcal{D}(f): \mathbb{R}, \quad f'(x) = -7(\sin(2-x)+1)^6\cos(2-x), \quad t: y = 15 - 7x.$