**Integrujte**

(1) $\int x \arcsin(x)\,\mathrm{d}x.$

(2) $\int x^2\sqrt{1-x^2}\,\mathrm{d}x.$

(3) $\int \sin^2 x\,\mathrm{d}x.$

(4) $\int \frac{1+\sqrt{x}}{x\left(\sqrt[3]{x}-1\right)}\,\mathrm{d}x.$

(5) $\int \frac{\ln^2\left(\ln(x)\right)}{x}\,\mathrm{d}x.$

(6) $\int \frac{x}{x-\sqrt{x-1}-1}\,\mathrm{d}x.$

(7) $\int \frac{\sqrt[3]{x+1}-1}{2\sqrt[3]{x+1}+1}\,\mathrm{d}x.$

(8) $\int \sqrt{x}\sin\left(\sqrt{x}\right)\mathrm{d}x.$

(9) $\int \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x.$

(10) $\int \frac{\sin(2x)}{2\cos^2(x)-3\sin(x)}\,\mathrm{d}x.$

(11) $\int \frac{\sqrt{x+1}+1}{\sqrt[3]{x+1}+1}\,\mathrm{d}x.$

(12) $\int \frac{(\ln(x)-1)^3}{x\left(\ln^2(x)+1\right)}\,\mathrm{d}x.$

(13) $\int \mathrm{e}^{2x}\arcsin\left(\mathrm{e}^x\right)\mathrm{d}x.$

(14) $\int \frac{1}{x^3\sqrt{x^2+1}}\,\mathrm{d}x.$

(15) $\int \ln\left(\sqrt{x}\right)\mathrm{d}x.$

(16) $\int \sqrt{1-x^2}\,\mathrm{d}x.$

(17) $\int \frac{\mathrm{e}^{3x}}{\mathrm{e}^{2x}+2\mathrm{e}^x-3}\,\mathrm{d}x.$

(18) $\int x^3\ln\left(x^2+1\right)\mathrm{d}x.$

(19) $\int \frac{x}{x+4\sqrt{x-1}+4}\,\mathrm{d}x.$

(20) $\int \frac{\mathrm{e}^x+\mathrm{e}^{-x}}{\mathrm{e}^x-\mathrm{e}^{-x}}\,\mathrm{d}x.$

(21) $\int \operatorname{arctg}\left(\sqrt[3]{x}\right)\mathrm{d}x.$

(22) $\int \sin\left(\sqrt{x}\right)\mathrm{d}x.$

(23) $\int \frac{1+\mathrm{e}^{2x}}{2-\mathrm{e}^{-x}}\,\mathrm{d}x.$

(24) $\int \frac{\mathrm{e}^{3x}}{\mathrm{e}^{2x}+2\mathrm{e}^x+5}\,\mathrm{d}x.$

(25) $\int \frac{\sin x+\sin^3 x}{\cos x+\cos^3 x}\,\mathrm{d}x.$

(26) $\int \frac{1}{x\sqrt{x^2+1}}\,\mathrm{d}x.$

(27) $\int \frac{1}{x-\sqrt{x}-2}\,\mathrm{d}x.$

(28) $\int \frac{x}{\cos^2 x}\,\mathrm{d}x.$

(29) $\int \arcsin x\,\mathrm{d}x.$

(30) $\int \frac{\sin(2x)}{\sin(x)-\cos(2x)}\,\mathrm{d}x.$

**Integrujte**

(1)
$$\int x \arcsin(x)\,\mathrm{d}x = \frac{x^2}{2}\arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + c = \frac{1}{4}\left(x\sqrt{1-x^2} + \left(2x^2-1\right)\arcsin(x)\right) + c$$

(2)
$$\int x^2\sqrt{1-x^2}\,\mathrm{d}x = \frac{1}{8}\arcsin x - \frac{1}{8}x\sqrt{1-x^2} + \frac{1}{4}x^3\sqrt{1-x^2} = \frac{1}{8}\left(x\sqrt{1-x^2}\left(2x^2-1\right) + \arcsin(x)\right) + c$$

(3) $\displaystyle\int \sin^2 x\,\mathrm{d}x = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c.$

(4) $\displaystyle\int \frac{1+\sqrt{x}}{x\left(\sqrt[3]{x}-1\right)}\,\mathrm{d}x = 6\left(\sqrt[6]{x} - \ln\left|\sqrt[6]{x}\right| + \ln\left|\sqrt[6]{x}-1\right|\right) + c.$

(5) $\displaystyle\int \frac{\ln^2\left(\ln(x)\right)}{x}\,\mathrm{d}x = \ln(x)\,\ln^2(\ln(x)) - 2\ln(x)\,\ln(\ln(x)) + 2\ln(x) + c.$

(6) $\displaystyle\int \frac{x}{x-\sqrt{x-1}-1}\,\mathrm{d}x = x - 1 + 2\sqrt{x-1} + 4\ln\left|-1+\sqrt{x-1}\right| + c.$

(7) $\displaystyle\int \frac{\sqrt[3]{x+1}-1}{2\sqrt[3]{x+1}+1}\,\mathrm{d}x = \frac{x+1}{2} - \frac{9}{8}\sqrt[3]{(x+1)^2} + \frac{9}{8}\sqrt[3]{x+1} - \frac{9}{16}\ln\left|2\sqrt[3]{x+1}\right| + c.$

(8) $\displaystyle\int \sqrt{x}\sin\left(\sqrt{x}\right)\mathrm{d}x = 4\sqrt{x}\sin\left(\sqrt{x}\right) - 2(x-2)\cos\left(\sqrt{x}\right) + c.$

(9) $\displaystyle\int \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x = 2\left(\sqrt{x}-1\right)\mathrm{e}^{\sqrt{x}} + c.$

(10) $\displaystyle\int \frac{\sin(2x)}{2\cos^2(x) - 3\sin(x)} = -\frac{1}{5}\ln\left|2\sin(x)-1\right| - \frac{4}{5}\ln\left|\sin(x)+2\right| + c.$

(11)
$$\begin{aligned}\int \frac{\sqrt{x+1}+1}{2\sqrt[3]{x+1}+1}\,\mathrm{d}x &= \\ &= 6\left(\frac{\sqrt[6]{(x+1)^7}}{7} - \frac{\sqrt[6]{(x+1)^5}}{5} + \frac{\sqrt[6]{(x+1)^4}}{4} - \frac{\sqrt[6]{(x+1)^3}}{3} - \frac{\sqrt[6]{(x+1)^2}}{2} - \sqrt[6]{x+1} + \right. \\ &\left. + \frac{1}{2}\ln\left|\sqrt[6]{(x+1)^2}+1\right| + \operatorname{arctg}\left(\sqrt[6]{x+1}\right)\right) + c.\end{aligned}$$

(12) $$\int \frac{(\ln(x)-1)^3}{x\left(\ln^2(x)+1\right)}\,\mathrm{d}x = \frac{\ln^2(x)}{2} - 3\ln(x) + \ln\left|\ln^2(x)+1\right| + 2\operatorname{arctg}\left(\ln(x)\right) + c.$$

(13) $$\begin{aligned}\int \mathrm{e}^{2x}\arcsin\left(\mathrm{e}^x\right)\mathrm{d}x &= \frac{\mathrm{e}^{2x}}{2}\arcsin\mathrm{e}^x - \frac{1}{4}\arcsin\mathrm{e}^x + \frac{1}{4}\mathrm{e}^x\sqrt{1-\mathrm{e}^{2x}} + c = \\ &= \frac{1}{4}\left(\mathrm{e}^x\sqrt{1-\mathrm{e}^{2x}} + \left(2\mathrm{e}^{2x}-1\right)\arcsin\left(\mathrm{e}^x\right)\right) + c\end{aligned}$$

(14) $$\int \frac{1}{x^3\sqrt{x^2+1}}\,\mathrm{d}x = \frac{1}{4}\left(\ln\left|1+\sqrt{x^2+1}\right| - \frac{1}{1+\sqrt{x^2+1}} - \ln\left|-1+\sqrt{x^2+1}\right| - \frac{1}{-1+\sqrt{x^2+1}}\right) + c.$$

(15) $$\int \ln\left(\sqrt{x}\right)\mathrm{d}x = x\ln\sqrt{x} - \frac{x}{2} + c = \frac{1}{2}(x\ln(x)-x) + c.$$

(16) $$\int \sqrt{1-x^2}\,\mathrm{d}x = \frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right) + c$$

(17) $$\int \frac{\mathrm{e}^{3x}}{\mathrm{e}^{2x}+2\mathrm{e}^x-3}\,\mathrm{d}x = \mathrm{e}^x - \frac{9}{4}\ln\left|\mathrm{e}^x+3\right| + \frac{1}{4}\ln\left|\mathrm{e}^x-1\right| + c.$$

(18) $$\begin{aligned}\int x^3\,\ln\left(x^2+1\right)\mathrm{d}x &= \frac{1}{4}\left(x^4-1\right)\ln\left(x^2+1\right) - \frac{1}{8}\left(x^4-2x^2-3\right) + c = \\ &= \frac{1}{4}\left(x^4-1\right)\ln\left(x^2+1\right) - \frac{x^4}{8} + \frac{x^2}{4} + c..\end{aligned}$$

(19) $$\int \frac{x}{x+4\sqrt{x-1}+4}\,\mathrm{d}x = 2\left(-4\sqrt{x-1} + \frac{x-1}{2} + 6\ln\left|x+4\sqrt{x-1}+4\right| - 4\operatorname{arctg}\left(2+\sqrt{x-1}\right)\right) + c.$$

(20) $$\int \frac{\mathrm{e}^x+\mathrm{e}^{-x}}{\mathrm{e}^x-\mathrm{e}^{-x}}\,\mathrm{d}x = -x + \ln\left|\mathrm{e}^{2x}-1\right| + c.$$

(21) $$\int \operatorname{arctg}\left(\sqrt[3]{x}\right)\mathrm{d}x = -\frac{\sqrt[3]{x^2}}{2} + \frac{1}{2}\ln\left(\sqrt[3]{x^2}+1\right) + x\operatorname{arctg}\left(\sqrt[3]{x}\right) + c.$$

(22) $$\int \sin\left(\sqrt{x}\right)\mathrm{d}x = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right) + c.$$

(23) $$\int \frac{1+\mathrm{e}^{2x}}{2-\mathrm{e}^{-x}}\,\mathrm{d}x = \frac{1}{8}\left(2\mathrm{e}^x + 2\mathrm{e}^{2x} + 5\ln\left|2\mathrm{e}^x-1\right|\right) + c.$$

(24) $$\int \frac{\mathrm{e}^{3x}}{\mathrm{e}^{2x}+2\mathrm{e}^x+5}\,\mathrm{d}x = \mathrm{e}^x - \ln\left|\mathrm{e}^{2x}+2\mathrm{e}^x+5\right| + \frac{3}{2}\operatorname{arctg}\left(\frac{\mathrm{e}^x+1}{2}\right) + c.$$

(25) $\displaystyle\int \frac{\sin x + \sin^3 x}{\cos x + \cos^3 x}\,\mathrm{d}x = -2\ln|\cos x| + \frac{3}{2}\ln\left|\cos^2 x + 1\right| + c.$

(26) $\displaystyle\int \frac{1}{x\sqrt{x^2+1}}\,\mathrm{d}x = \frac{1}{2}\left(-\ln\left|\sqrt{x^2+1}+1\right| + \ln\left|\sqrt{x^2+1}-1\right|\right) + c.$

(27) $\displaystyle\int \frac{1}{x-\sqrt{x}-2}\,\mathrm{d}x = \frac{2}{3}\ln\left|1+\sqrt{x}\right| + \frac{4}{3}\ln\left|-2+\sqrt{x}\right| + c.$

(28) $\displaystyle\int \frac{x}{\cos^2 x}\,\mathrm{d}x = x\ \mathrm{tg}\,x + \ln|\cos x| + c.$

(29) $\displaystyle\int \arcsin x\,\mathrm{d}x = x\arcsin x + \sqrt{1-x^2} + c.$

(30) $\displaystyle\frac{\sin(2x)}{\sin(x) - \cos(2x)} = \frac{1}{3}\ln\left|2\sin(x) - 1\right| + \frac{2}{3}\ln\left|\sin(x) + 2\right| + c.$