Určete a zakreslete definiční obor $f(x,y)$, spočtěte derivaci :

$$f\,(x,y) = \ln\left(1 - x^2 - 4y^2\right) + \sqrt{4x^2 + y^2 - 1}, \qquad \frac{\partial f}{\partial x}.$$

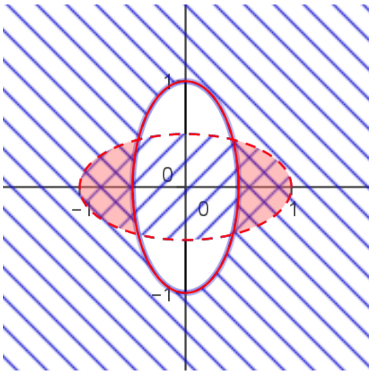
Řešení: Argument logaritmické funkce musí být vždy kladný, proto vyžadujeme splnění podmínky

$$1 - x^2 - 4y^2 > 0 \Rightarrow 1 > x^2 + 4y^2.$$

Rovnice $1 = x^2 + 4y^2$ je rovnicí elipsy se středem v počátku, s hlavní poloosou ležící na ose x. Délka hlavní poloosy je 1 a délky poloosy vedlejší je $\frac{1}{2}$. Uvedenou podmínku pak splňují všechny body „uvnitř elipsy.“
Podmínka pro $\sqrt{4x^2 + y^2 - 1}$ je obdobná,

$$4x^2 + y^2 - 1 \geq 0 \Rightarrow 4x^2 + y^2 \geq 1.$$

I zde se jedná o elipsu, tentokráte s hlavní poloosou na ose y. Podmínku splňují všechny body „vně elipsy.“ Protože však v podmínce připouštíme rovnost, je podmínka splněna i body samotné elipsy.



Obrázek 1: Náčrtek definičního oboru fce

Na obrázku 1 je oblast, na níž je funkce $f(x,y)$ definována, zakreslena červenou barvou. Definiční obor můžeme samozřejmě zapsat také množinově, tj.

$$\mathcal{D}(f) = \left\{[x,y] \in \mathbb{R}^2 \;|1 > x^2 + 4y^2 \wedge 4x^2 + y^2 \geq 1\right\}.$$

Zbývá spočíst naznačenou parciální derivaci

$$\frac{\partial f}{\partial x} = \frac{\partial \ln\left(1 - x^2 - 4y^2\right) + \sqrt{4x^2 + y^2 - 1}}{\partial x} = \frac{\partial \ln\left(1 - x^2 - 4y^2\right)}{\partial x} + \frac{\partial \sqrt{4x^2 + y^2 - 1}}{\partial x} =$$
$$= \frac{1}{1 - x^2 - 4y^2}\;\frac{\partial\left(1 - x^2 - 4y^2\right)}{\partial x} + \frac{1}{2\sqrt{4x^2 + y^2 - 1}}\;\frac{\partial(4x^2 + y^2 - 1)}{\partial x} =$$
$$= \frac{1}{1 - x^2 - 4y^2}\;(-2x) + \frac{1}{2\sqrt{4x^2 + y^2 - 1}}\;(8x) =$$
$$= \frac{-2x}{1 - x^2 - 4y^2} + \frac{4x}{\sqrt{4x^2 + y^2 - 1}}.$$

Určete a zakreslete definiční obor $f(x,y)$, spočtěte derivaci :

(1) $f(x,y) = \ln\Big(x - y^2 + 1\Big)\,, \qquad \dfrac{\partial f}{\partial y}\,.$

(2) $f(x,y) = \sqrt{x^2 - y^2}, \qquad \dfrac{\partial f}{\partial x}\,.$

(3) $f(x,y) = \arcsin\Big(x^2 + y^2 - 3\Big)\,, \qquad \dfrac{\partial f}{\partial y}\,.$

(4) $f(x,y) = \dfrac{y}{x^2 - 1}, \qquad \dfrac{\partial f}{\partial x}\,.$

(5) $f(x,y) = \sqrt{1 - x\,y}, \qquad \dfrac{\partial f}{\partial y}\,.$

(6) $f(x,y) = \ln\left(1 - 2\,x - 3\,y\right), \qquad \dfrac{\partial f}{\partial x}\,.$

(7) $f(x,y) = \ln xy, \qquad \dfrac{\partial f}{\partial x}\,.$

(8) $f(x,y) = \arcsin\dfrac{y-1}{x}, \qquad \dfrac{\partial f}{\partial y}\,.$

(9) $f(x,y) = \arccos\left(\dfrac{x}{y^2}\right), \qquad \dfrac{\partial f}{\partial x}\,.$

(10) $f(x,y) = \sqrt{4x - y^2}, \qquad \dfrac{\partial f}{\partial x}\,.$

(11) $f(x,y) = \sqrt{1 - (x^2 - y)^2}, \qquad \dfrac{\partial f}{\partial y}\,.$

(12) $f(x,y) = \arcsin(1 - x^2 - y^2), \qquad \dfrac{\partial f}{\partial x}\,.$

(13) $f(x,y) = \arctan\dfrac{2y}{x^2 + y^2 - 1}, \qquad \dfrac{\partial f}{\partial x}\,.$

(14) $f(x,y) = \sqrt{\dfrac{y-1}{x}}, \qquad \dfrac{\partial f}{\partial x}\,.$

(15) $f(x,y) = \dfrac{xy}{x^2 + y^2 - 1}, \qquad \dfrac{\partial f}{\partial x}\,.$

(16) $f\left(x,y\right) = \ln\Big(y^2 - x^2\Big)\,, \qquad \dfrac{\partial f}{\partial x}\,.$

(17) $f\left(x,y\right) = \dfrac{xy}{x - y}, \qquad \dfrac{\partial f}{\partial y}\,.$

(18)
$f\left(x,y\right) = \ln\left(x - 2y\right) - \sqrt{y - 2x}, \qquad \dfrac{\partial f}{\partial x}\,.$

(19) $f\left(x,y\right) = \ln\left(x + xy\right), \qquad \dfrac{\partial f}{\partial y}\,.$

(20) $f\left(x,y\right) = \dfrac{1}{\sqrt{1 + y^2 - x}}, \qquad \dfrac{\partial f}{\partial y}\,.$

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Určete a zakreslete definiční obor $f(x, y)$, spočtěte derivaci :

(1) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid x > y^2 - 1\Big\}, \quad \dfrac{\partial f}{\partial y} = \dfrac{-2y}{x - y^2 + 1}.$

(2) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid (x \geq 0 \wedge -x < y < x) \vee (x < 0 \wedge x < y < -x)\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{x}{\sqrt{x^2 - y^2}}.$

(3) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid 2 \leq x^2 + y^2 \leq 4\Big\}, \quad \dfrac{\partial f}{\partial y} = \dfrac{2y}{\sqrt{1 - (x^2 + y^2 - 3)^2}}.$

(4) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid x \neq \pm 1\Big\}, \quad \dfrac{\partial f}{\partial x} = -\Big(x^2 - 1\Big)^{-2} 2xy = \dfrac{-2xy}{(x^2 - 1)^2}.$

(5) $\mathcal{D}(f) = \left\{[x, y] \in \mathbb{R}^2 \mid \left(x \geq 0 \wedge y \leq \dfrac{1}{x}\right) \vee \left(x < 0 \wedge \dfrac{1}{x} < y\right)\right\}, \quad \dfrac{\partial f}{\partial y} = -\dfrac{x}{2\sqrt{1 - x\, y}}.$

(6) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid 1 - 2\,x - 3\,y > 0\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{-2}{1 - 2\,x - 3\,y}.$

(7) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid (x > 0 \wedge y > 0) \vee (x < 0 \wedge y < 0)\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{y}{xy} = \dfrac{1}{y}.$

(8)
$\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid (x < 0 \wedge 1 + x \leq y \leq 1 - x) \vee (x > 0 \wedge 1 - x \leq y \leq 1 + x)\Big\}, \quad \dfrac{\partial f}{\partial y} = \dfrac{1}{x\sqrt{1 - \left(\frac{y-1}{x}\right)^2}}$

(9) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid -y^2 \leq x \leq y^2,\ y \neq 0\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{-1}{y^2\sqrt{1 - \left(\frac{x}{y^2}\right)^2}}.$

(10) $\mathcal{D}(f) = \left\{[x, y] \in \mathbb{R}^2 \mid x \geq \dfrac{1}{4}y^2\right\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{2}{\sqrt{4x - y^2}}.$

(11) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid x^2 - 1 \leq y \leq x^2 + 1\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{(x^2 - y)}{\sqrt{1 - (x^2 - y)^2}}.$

(12) $\mathcal{D}(f) = \Big\{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 \leq 2\Big\}, \quad \dfrac{\partial f}{\partial x} = \dfrac{-2x}{\sqrt{1 - (1 - x^2 - y^2)^2}}.$

(13) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid x^2+y^2\neq 1\right\},\quad \dfrac{\partial f}{\partial x}=\dfrac{-4xy}{\left(1+\left(\frac{2y}{x^2+y^2-1}\right)^2\right)(x^2+y^2-1)^2}.$

(14) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid y\geq 1-x,\ y\leq x+1,\ x\neq 0\right\},\quad \dfrac{\partial f}{\partial x}=\dfrac{y-1}{x^2\sqrt{1-\left(\frac{y-1}{x}\right)^2}}.$

(15) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid x^2+y^2\neq 1\right\},\quad \dfrac{\partial f}{\partial x}=\dfrac{-y\left(x^2-y^2+1\right)}{\left(x^2+y^2-1\right)^2}.$

(16) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid (y<0\wedge -y<x<y)\vee(y>0\wedge y<x<-y)\right\},\quad \dfrac{\partial f}{\partial x}=-\dfrac{2x}{y^2-x^2}.$

(17) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid x\neq y\right\},\quad \dfrac{\partial f}{\partial y}=\dfrac{x^2}{(x-y)^2}.$

(18) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid 2x\leq y<\dfrac{x}{2}\right\},\quad \dfrac{\partial f}{\partial x}=\dfrac{1}{x-2y}+\dfrac{1}{\sqrt{y-2x}}.$

(19) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid (x<0\wedge y<-1)\vee(x>0\wedge y>-1)\right\},\quad \dfrac{\partial f}{\partial y}=\dfrac{1}{1+y}.$

(20) $\mathcal{D}(f)=\left\{[x,y]\in\mathbb{R}^2 \mid 1+y^2>x\right\},\quad \dfrac{\partial f}{\partial y}=-\dfrac{y}{\sqrt{(1+y^2-x)^3}}.$