

Určete definiční obor fce $f(x)$ a limity v jeho krajních bodech.

1.
$$f(x) = \log \left(\log \left(\log (1 - x) \right) \right)$$

2.
$$f(x) = \sqrt{\arcsin \left(\frac{1 - x}{3x - 1} \right)}$$

3.
$$f(x) = \ln \left(\frac{x^2 - 1}{x^2 - 4x + 4} \right)$$

4.
$$f(x) = \operatorname{arctg} \left(\frac{x^2 - 4}{16 - x^2} \right)$$

5.
$$f(x) = \operatorname{arctg} \sqrt{3x^2 - x - 4}$$

6.
$$f(x) = \arcsin \sqrt{\frac{x - 2}{x^2 + 4}}$$

7.
$$f(x) = \arcsin \left(x + \sqrt{x^2 + 3x + 11} \right)$$

8.
$$f(x) = \sqrt{3 - \ln(x + 1)}$$

9.
$$f(x) = \ln \left(\frac{x^2 - 3x + 2}{x^2 - 7x + 12} \right)$$

10.
$$f(x) = \frac{1}{1 - \sqrt{1 - x^2}}$$

11.
$$f(x) = \sqrt{\ln(x^2 - 3x - 3)}$$

12.
$$f(x) = \sqrt{\ln \left(\frac{x - 4}{x^2 - 4} \right)}$$

13.
$$f(x) = \frac{1}{\sqrt{\arccos(2x^2 + 5x + 1)}}$$

14.
$$f(x) = \arcsin \left(\frac{x - 5}{x^2 - 1} \right)$$

15.
$$f(x) = \ln \left(x + \sqrt{x^2 + 3x + 2} \right)$$

16.
$$f(x) = \ln \left(\operatorname{arccotg} \left(x^3 - x^{-3} \right) \right)$$

17.
$$f(x) = \ln \left(\arcsin \left(x^2 + 3x + 3 \right) \right)$$

18.
$$f(x) = \operatorname{arctg} \left(\sqrt{12 + x - x^2} \right)$$

19.
$$f(x) = \sqrt{\operatorname{arctg}(x^2 + x - 6)}$$

20.
$$f(x) = \ln \left(\ln \left(x^2 + 3x + 3 \right) \right)$$

21.
$$f(x) = \arcsin \left(\frac{1}{1 - x^2} \right)$$

22.
$$f(x) = \sqrt{\ln(x^2 + x - 1)}$$

23.
$$f(x) = \arccos \left(\frac{1 - x}{x^2 + 5x + 6} \right)$$

24.
$$f(x) = \ln \left(\frac{x + 2}{1 - x^2} \right)$$

25.
$$f(x) = \ln \left(\frac{x^2 + 2x - 3}{x^2 - 2x - 3} \right)$$

26.
$$f(x) = \arccos(-x^2 - x + 1)$$

27.
$$f(x) = \ln \left(\ln \left(\frac{x}{x + 2} \right) \right)$$

28.
$$f(x) = \sqrt{\ln(5 - \sqrt{x - 5})}$$

29.
$$f(x) = \ln \sqrt{1 - \sqrt{x - 4}}$$

30.
$$f(x) = \frac{1}{1 - \sqrt{x^2 - 4x + 4}}$$

Určete definiční obor fce $f(x)$ a limity v jeho krajních bodech.

1.

$$\mathcal{D}(f) = (-\infty, -9), \quad \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow -9^-} f(x) = -\infty,$$

2.

$$\mathcal{D}(f) = \left\langle \frac{1}{2}, 1 \right\rangle \quad \lim_{x \rightarrow \frac{1}{2}^+} f(x) = \sqrt{\frac{\pi}{2}}, \quad \lim_{x \rightarrow 1^-} f(x) = 0,$$

3.

$$\mathcal{D}(f) = (-\infty, -1) \cup (1, 2) \cup (2, \infty),$$

$$\lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow -1^-} f(x) = -\infty, \quad \lim_{x \rightarrow 1^+} f(x) = -\infty, \quad \lim_{x \rightarrow 2^-} f(x) = \infty, \quad \lim_{x \rightarrow 2^+} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = 0,$$

4.

$$\mathcal{D}(f) = (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$$

$$\lim_{x \rightarrow -\infty} f(x) = -\frac{\pi}{4}, \quad \lim_{x \rightarrow -4^-} f(x) = -\frac{\pi}{2}, \quad \lim_{x \rightarrow -4^+} f(x) = \frac{\pi}{2}, \quad \lim_{x \rightarrow 4^-} f(x) = \frac{\pi}{2}, \quad \lim_{x \rightarrow 4^+} f(x) = -\frac{\pi}{2}, \quad \lim_{x \rightarrow \infty} f(x) = -\frac{\pi}{4},$$

5.

$$\mathcal{D}(f) = (-\infty, -1) \cup \left\langle \frac{4}{3}, \infty \right\rangle, \quad \lim_{x \rightarrow -\infty} f(x) = \frac{\pi}{2}, \quad \lim_{x \rightarrow -1^-} f(x) = 0, \quad \lim_{x \rightarrow \frac{4}{3}^+} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = \frac{\pi}{2}$$

6.

$$\mathcal{D}(f) = \langle 2, \infty \rangle \quad \lim_{x \rightarrow 2^+} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = 0,$$

7.

$$\mathcal{D}(f) = \langle -10, -2 \rangle, \quad \lim_{x \rightarrow -10^+} f(x) = -\frac{\pi}{2}, \quad \lim_{x \rightarrow -2^-} f(x) = \frac{\pi}{2},$$

8.

$$\mathcal{D}(f) = \left\langle -1, -1 + e^3 \right\rangle, \quad \lim_{x \rightarrow -1^+} f(x) = \infty, \quad \lim_{x \rightarrow (-1+e^3)^-} f(x) = 0,$$

9.

$$\mathcal{D}(f) = (-\infty, 1) \cup (2, 3) \cup (4, \infty)$$

$$\lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow 1^-} f(x) = -\infty, \quad \lim_{x \rightarrow 2^+} f(x) = -\infty, \quad \lim_{x \rightarrow 3^-} f(x) = \infty, \quad \lim_{x \rightarrow 4^+} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = 0$$

10.

$$\mathcal{D}(f) = \langle -1, 0 \rangle \cup (0, 1), \quad \lim_{x \rightarrow -1^+} f(x) = 1, \quad \lim_{x \rightarrow 0^-} f(x) = \infty, \quad \lim_{x \rightarrow 0^+} f(x) = \infty, \quad \lim_{x \rightarrow 1^-} f(x) = 1,$$

11.

$$\mathcal{D}(f) = (-\infty, -1) \cup \langle 4, \infty \rangle \quad \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow -1^-} f(x) = 0, \quad \lim_{x \rightarrow 4^+} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

12.

$$\mathcal{D}(f) = (-2, 0) \cup \langle 1, 2 \rangle \quad \lim_{x \rightarrow -2^+} f(x) = \infty, \quad \lim_{x \rightarrow 0^-} f(x) = 0, \quad \lim_{x \rightarrow 1^+} f(x) = 0, \quad \lim_{x \rightarrow 2^-} f(x) = \infty,$$

$$13. \quad \mathcal{D}(f) = \left(-\frac{5}{2}, -2\right) \cup \left(-\frac{1}{2}, 0\right) \quad \lim_{x \rightarrow -\frac{5}{2}^+} f(x) = \infty, \quad \lim_{x \rightarrow -2^-} f(x) = \frac{1}{\sqrt{\pi}}, \quad \lim_{x \rightarrow -\frac{1}{2}^+} f(x) = \frac{1}{\sqrt{\pi}}, \quad \lim_{x \rightarrow 0^-} f(x) = \infty$$

$$14. \quad \mathcal{D}(f) = (-\infty, -3) \cup (2, \infty), \quad \lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow -3^-} f(x) = -\frac{\pi}{2}, \quad \lim_{x \rightarrow 2^+} f(x) = -\frac{\pi}{2}, \quad \lim_{x \rightarrow \infty} f(x) = 0,$$

$$15. \quad \mathcal{D}(f) = \left(-\frac{2}{3}, \infty\right), \quad \lim_{x \rightarrow -\frac{2}{3}^+} f(x) = -\infty, \quad \lim_{x \rightarrow \infty} f(x) = \infty,$$

$$16. \quad \mathcal{D}(f) = \mathbb{R} \setminus \{0\}, \quad \lim_{x \rightarrow -\infty} f(x) = \ln \pi, \quad \lim_{x \rightarrow 0^-} f(x) = -\infty, \quad \lim_{x \rightarrow 0^+} f(x) = \ln \pi, \quad \lim_{x \rightarrow \infty} f(x) = -\infty,$$

$$17. \quad \mathcal{D}(f) = \langle -2, -1 \rangle \quad \lim_{x \rightarrow -2^+} f(x) = \ln\left(\frac{\pi}{2}\right), \quad \lim_{x \rightarrow 1^-} f(x) = \ln\left(\frac{\pi}{2}\right)$$

$$18. \quad \mathcal{D}(f) = \langle -3, 4 \rangle \quad \lim_{x \rightarrow -3^+} f(x) = 0, \quad \lim_{x \rightarrow 4^-} f(x) = 0,$$

$$19. \quad \mathcal{D}(f) = (-\infty, -3) \cup (2, \infty) \quad \lim_{x \rightarrow -\infty} f(x) = \sqrt{\frac{\pi}{2}}, \quad \lim_{x \rightarrow -3^-} f(x) = 0, \quad \lim_{x \rightarrow 2^+} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = \sqrt{\frac{\pi}{2}}$$

$$20. \quad \mathcal{D}(f) = (-\infty, -2) \cup (-1, \infty), \quad \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow -2^-} f(x) = -\infty, \quad \lim_{x \rightarrow -1^+} f(x) = -\infty, \quad \lim_{x \rightarrow \infty} f(x) = \infty,$$

$$21. \quad \mathcal{D}(f) = \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right) \\ \lim_{x \rightarrow -\infty} f(x) = 0^-, \quad \lim_{x \rightarrow -\sqrt{2}^-} f(x) = \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}, \quad \lim_{x \rightarrow \sqrt{2}^+} f(x) = \frac{\pi}{4}, \quad \lim_{x \rightarrow \infty^-} f(x) = 0^+$$

$$22. \quad \mathcal{D}(f) = (-\infty, -2) \cup (1, \infty) \quad \lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow -2^-} f(x) = 0, \quad \lim_{x \rightarrow 1^+} f(x) = 0, \quad \lim_{x \rightarrow 1^-} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

$$23. \quad \mathcal{D}(f) = (-\infty, -5) \cup (-1, \infty) \quad \lim_{x \rightarrow -\infty} f(x) = \frac{\pi}{2}, \quad \lim_{x \rightarrow -5^-} f(x) = 0, \quad \lim_{x \rightarrow -1^+} f(x) = 0, \quad \lim_{x \rightarrow \infty} f(x) = 0$$

$$24. \quad \mathcal{D}(f) = (-\infty, -2) \cup (-1, 1) \quad \lim_{x \rightarrow -\infty} f(x) = -\infty, \quad \lim_{x \rightarrow -2^-} f(x) = -\infty, \quad \lim_{x \rightarrow -1^+} f(x) = \infty, \quad \lim_{x \rightarrow 1^-} f(x) = \infty,$$

$$25. \quad \mathcal{D}(f) = (-\infty, -3) \cup (-1, 1) \cup (3, \infty), \\ \lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow -3^-} f(x) = -\infty, \quad \lim_{x \rightarrow -1^+} f(x) = \infty, \quad \lim_{x \rightarrow 1^-} f(x) = -\infty, \quad \lim_{x \rightarrow 3^+} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = 0$$

26.

$$\mathcal{D}(f) = \langle -2, -1 \rangle \cup \langle 0, 1 \rangle \quad \lim_{x \rightarrow -2^+} f(x) = \pi, \quad \lim_{x \rightarrow -1^-} f(x) = 0, \quad \lim_{x \rightarrow 0^+} f(x) = 0, \quad \lim_{x \rightarrow 1^-} f(x) = \pi, \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

27.

$$\mathcal{D}(f) = (-\infty, -2) \quad \lim_{x \rightarrow -\infty} f(x) = -\infty, \quad \lim_{x \rightarrow -2^-} f(x) = \infty$$

28.

$$\mathcal{D}(f) = \langle 5, 21 \rangle, \quad \lim_{x \rightarrow 5^+} f(x) = \sqrt{\ln 5}, \quad \lim_{x \rightarrow 21^-} f(x) = 0$$

29.

$$\mathcal{D}(f) = \langle 4, 5 \rangle, \quad \lim_{x \rightarrow 4^+} f(x) = 0, \quad \lim_{x \rightarrow 5^-} f(x) = -\infty$$

30.

$$\mathcal{D}(f) = (-\infty, 1) \cup (1, 3) \cup (3, \infty),$$

$$\lim_{x \rightarrow -\infty} f(x) = 0, \quad \lim_{x \rightarrow 1^-} f(x) = -\infty, \quad \lim_{x \rightarrow 1^+} f(x) = \infty, \quad \lim_{x \rightarrow 3^-} f(x) = \infty, \quad \lim_{x \rightarrow 3^+} f(x) = -\infty, \quad \lim_{x \rightarrow \infty} f(x) = 0,$$