2016/2017

Určete definiční obor fce $f(x)$ a spočtěte derivaci $f'(x)$.

1. $f(x) = \dfrac{\cos 2\,x}{\cos x}$

2. $f(x) = \ln\left(\dfrac{1}{1-x^2}\right)$

3. $f(x) = \operatorname{arctg}\left(\dfrac{x}{x^2-1}\right)$

4. $f(x) = \dfrac{x}{\ln x}$

5. $f(x) = \dfrac{\ln\left(\frac{1}{x+2}\right)}{x+2}$

6. $f(x) = \sqrt{\cos\left(-2\,x\right)}$

7. $f(x) = \sqrt{\dfrac{1}{4-x^2}}$

8. $f(x) = \arccos\left(\dfrac{1}{x-1}\right)$

9. $f(x) = \dfrac{1}{\ln(x^2)-1}$

10. $f(x) = \sqrt{x\left(4-x^2\right)}$

11. $f(x) = \sqrt{1+\dfrac{x}{1-x}}$

12. $f(x) = \dfrac{\cos 2\,x}{\sin x}$

13. $f(x) = \sin\left(1+\sqrt{1-x}\right)$

14. $f(x) = \dfrac{\cos x}{\mathrm{e}^{\cos x}}$

15. $f(x) = \ln\left(1-\ln x\right)$

16. $f(x) = \sqrt{x^2+\sqrt{-x}}$

17. $f(x) = \ln\left(\dfrac{x^2}{x-1}\right)$

18. $f(x) = \ln\left(\sqrt{1-x^2}\right)$

19. $f(x) = \ln\left(\dfrac{1-2x}{x+2}\right)$

20. $f(x) = \sin\left(\ln\left(1-x^2\right)\right)$

21. $f(x) = \dfrac{\ln(\cos x)}{\operatorname{cotg} x}$

22. $f(x) = \sin^{-2}\left(2\,x\right)$

23. $f(x) = \ln\left(\dfrac{1}{\cos\left(\frac{x}{2}\right)}\right)$

24. $f(x) = \sqrt{\ln\left(1+x^2\right)}$

25. $f(x) = \ln\left(1-\cos^2(x)\right)$

26. $f(x) = \dfrac{1}{\ln^2 x - 1}$

27. $f(x) = \arcsin\left(\sqrt{1-x}\right)$

28. $f(x) = \ln\left(1 + \sin^2\left(\frac{x}{2}\right)\right)$

29. $f(x) = \mathrm{e}^{\sqrt{-x}}$

30. $f(x) = \sqrt{1 - \cos(x)}$

31. $f(x) = x\,\mathrm{arctg}\left(1 - x^2\right)$

32. $f(x) = \dfrac{1 + \sqrt{x}}{\sqrt{1 - x}}$

33. $f(x) = \ln\left(\sqrt{x^3 - 1}\right)$

34. $f(x) = \dfrac{\mathrm{e}^{\cos x}}{\sin x}$

35. $f(x) = \sqrt{\ln\left(1 - x\right)}$

36. $f(x) = \ln\left(\sqrt{1 + x^2}\right)$

37. $f(x) = \dfrac{\sqrt{x + 1}}{1 - \sqrt{x}}$

38. $f(x) = \arcsin\left(\dfrac{1}{x} - 1\right)$

39. $f(x) = \sqrt{1 - \sin^2(2x)}$

40. $f(x) = \ln\left(\dfrac{x}{1 - x^2}\right)$

41. $f(x) = \dfrac{\ln x}{x}$

42. $f(x) = \dfrac{x}{\sqrt{x^2 - 1}}$

43. $f(x) = \dfrac{1}{x\sqrt{1 - x^2}}$

44. $f(x) = \sqrt{\dfrac{-x}{x^2 - 1}}$

45. $f(x) = \ln\left(\sin^2(2x)\right)$

46. $f(x) = x^2\,\mathrm{arctg}\left(x^{-1}\right)$

47. $f(x) = \sqrt{x}\ln\left(2 + x - x^2\right)$

48. $f(x) = \sin\left(\ln\left(x - x^2\right)\right)$

49. $f(x) = \sqrt{x}\left(4 - x^2\right)$

50. $f(x) = \cos\left(\sqrt{1 - x}\right)$

2016/2017

Určete definiční obor fce $f(x)$ a spočtěte derivaci $f'(x)$.

1. $\mathcal{D}(f) = \mathbb{R} \setminus \left\{ (2k+1)\frac{\pi}{2}, k \in \mathbb{Z} \right\}, \quad f'(x) = -\frac{2\sin 2x \, \cos x + \cos 2x \, \sin x}{\cos^2 x}.$

2. $\mathcal{D}(f) = (-1,1), \quad f'(x) = \frac{2x}{1-x^2}.$

3. $\mathcal{D}(f) = \mathbb{R} \setminus \{-1,1\}, \quad f'(x) = -\frac{x^2+1}{x^4-x^2+1}.$

4. $\mathcal{D}(f) = (0,1) \cup (1,\infty), \quad f'(x) = \frac{-1+\ln x}{\ln^2 x}.$

5. $\mathcal{D}(f) = (-2,\infty), \quad f'(x) = -\frac{1+\ln\left(\frac{1}{x+2}\right)}{(x+2)^2}.$

6. $\mathcal{D}(f) = \bigcup_{k\in\mathbb{Z}} \left\langle -\frac{\pi}{4}+k\pi, \frac{\pi}{4}+k\pi \right\rangle \quad f'(x) = -\frac{\sin(2x)}{\sqrt{\cos(2x)}}.$

7. $\mathcal{D}(f) = (-2,2), \quad f'(x) = \frac{x}{(4-x^2)^{3/2}}.$

8. $\mathcal{D}(f) = (-\infty,0\rangle \cup \langle 2,\infty), \quad f'(x) = \frac{1}{(x-1)^2\sqrt{1-\frac{1}{(x-1)^2}}}.$

9. $\mathcal{D}(f) = \mathbb{R} \setminus \left\{-\sqrt{\mathrm{e}}, 0, \sqrt{\mathrm{e}}\right\}, \quad f'(x) = -\frac{2}{x\left(\ln(x^2)-1\right)^2}.$

10. $\mathcal{D}(f) = (-\infty,-2\rangle \cup \langle 0,2\rangle, \quad f'(x) = \frac{4-3x^2}{2\sqrt{x(4-x^2)}}.$

11. $\mathcal{D}(f) = (-\infty,1), \quad f'(x) = \frac{1}{2(1-x)^2\sqrt{1+\frac{x}{1-x}}}$

12. $\mathcal{D}(f) = \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}, \quad f'(x) = -\frac{2\sin 2x \, \sin x + \cos 2x \, \cos x}{\sin^2 x}.$

13.
$$\mathcal{D}(f) = (-\infty, 1\rangle, \quad f'(x) = -\frac{\cos\left(1+\sqrt{1-x}\right)}{2\sqrt{1-x}}.$$

14.
$$\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = \frac{\sin(x)\left(\cos(x)-1\right)}{\mathrm{e}^{\cos(x)}}.$$

15.
$$\mathcal{D}(f) = (0, \mathrm{e}), \quad f'(x) = \frac{1}{x(\ln x - 1)}.$$

16.
$$\mathcal{D}(f) = (-\infty, 0\rangle, \quad f'(x) = \frac{2x - \frac{1}{2\sqrt{-x}}}{2\sqrt{x^2+\sqrt{-x}}}.$$

17.
$$\mathcal{D}(f) = (1, \infty), \quad f'(x) = \frac{x-2}{x(x-1)}.$$

18.
$$\mathcal{D}(f) = (-1, 1), \quad f'(x) = \frac{x}{x^2-1}.$$

19.
$$\mathcal{D}(f) = \left(-2, \frac{1}{2}\right), \quad f'(x) = \frac{5}{(x+2)(2x-1)}.$$

20.
$$\mathcal{D}(f) = (-1, 1), \quad f'(x) = -\frac{2x\cos\left(\ln\left(1-x^2\right)\right)}{1-x^2}.$$

21.
$$\mathcal{D}(f) = \bigcup_{k\in\mathbb{Z}}\left(\left(-\left(4k-1\right)\frac{\pi}{2}, 2k\pi\right) \cup \left(2k\pi, \left(4k+1\right)\frac{\pi}{2}\right)\right), \quad f'(x) = \frac{-1+\dfrac{\ln\left(\cos x\right)}{\sin^2 x}}{\operatorname{cotg}^2 x}$$

22.
$$\mathcal{D}(f) = \mathbb{R}\setminus\left\{\frac{k\pi}{2}, k\in\mathbb{Z}\right\}, \quad f'(x) = \frac{-4\cos\left(2x\right)}{\sin^3\left(2\,x\right)}.$$

23.
$$\mathcal{D}(f) = \bigcup_{k\in\mathbb{Z}}\left(-\pi+4k\pi, \pi+4k\pi\right) \quad f'(x) = \frac{1}{2}\operatorname{tg}\left(\frac{x}{2}\right).$$

24.
$$\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = \frac{x}{\left(1+x^2\right)\sqrt{\ln\left(1+x^2\right)}}.$$

25.
$$\mathcal{D}(f) = \mathbb{R}\setminus\left\{k\,\pi; k\in\mathbb{Z}\right\}, \quad f'(x) = \frac{2\sin(x)\cos(x)}{1-\cos^2(x)} = 2\operatorname{cotg}\left(x\right).$$

26. $\mathcal{D}(f) = \left(0, \frac{1}{\mathrm{e}}\right) \cup \left(\frac{1}{\mathrm{e}}, \mathrm{e}\right) \cup (\mathrm{e}, \infty)\,, \quad f'(x) = -\frac{2\ln x}{x\left(\ln^2 x - 1\right)^2}\,.$

27. $\mathcal{D}(f) = \langle 0, 1\rangle\,, \quad f'(x) = -\frac{1}{2\sqrt{x}\sqrt{1-x}}.$

28. $\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = \frac{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{1+\sin^2\left(\frac{x}{2}\right)}.$

29. $\mathcal{D}(f) = (-\infty, 0\rangle\,, \quad f'(x) = -\frac{\mathrm{e}^{\sqrt{-x}}}{2\sqrt{-x}}.$

30. $\mathcal{D}(f) = \mathbb{R} \quad f'(x) = \frac{\sin(x)}{2\sqrt{1-\cos(x)}}\,.$

31. $\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = \operatorname{arctg}\left(1-x^2\right) - \frac{2x^2}{1+\left(1-x^2\right)^2}\,.$

32. $\mathcal{D}(f) = \langle 0, 1)\,, \quad f'(x) = \frac{\frac{1}{\sqrt{x}}\sqrt{1-x} + \left(1+\sqrt{x}\right)\frac{1}{\sqrt{1-x}}}{2\left(1-x\right)}\,.$

33. $\mathcal{D}(f) = (1, \infty), \quad f'(x) = \frac{3x^2}{2\left(x^3-1\right)}.$

34. $\mathcal{D}(f) = \mathbb{R}\setminus\{k\,\pi; k \in \mathbb{Z}\}\,, \quad f'(x) == -\frac{\mathrm{e}^{\cos x}\left(\sin^2 x + \cos x\right)}{\sin^2 x}.$

35. $\mathcal{D}(f) = (-\infty, 0\rangle\,, \quad f'(x) = -\frac{1}{2(1-x)\sqrt{\ln(1-x)}}.$

36. $\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = \frac{x}{1+x^2}.$

37. $\mathcal{D}(f) = (0,1) \cup (1, \infty)\,, \qquad f'(x) = \frac{\frac{1}{\sqrt{x+1}}\left(1-\sqrt{x}\right) + \sqrt{x+1}\frac{1}{\sqrt{x}}}{2\left(1-\sqrt{x}\right)^2}$

38. $\mathcal{D}(f) = \left\langle \frac{1}{2}, \infty\right), \quad f'(x) = -\frac{1}{x^2\sqrt{1-\left(\frac{1}{x}-1\right)^2}}$

39. $\mathcal{D}(f) = \mathbb{R}, \quad f'(x) = -\dfrac{2\sin(2x)\cos(2x)}{\sqrt{1-\sin^2(2x)}}.$

40. $\mathcal{D}(f) = (-\infty,-1)\cup(0,1)\,, \quad f' = \dfrac{1+x^2}{x(1-x^2)}.$

41. $\mathcal{D}(f) = (0,\infty), \quad f'(x) = \dfrac{1-\ln x}{x^2}.$

42. $\mathcal{D}(f) = (-\infty,-1)\cup(1,\infty), \quad f'(x) = -\dfrac{1}{(x^2-1)^{3/2}}.$

43. $\mathcal{D}(f) = (-1,0)\cup(0,1), \quad f'(x) = -\dfrac{1-2x^2}{x^2\,(1-x^2)^{3/2}}.$

44. $\mathcal{D}(f) = (-\infty,-1)\cup\langle 0,1), \quad f'(x) = \dfrac{x^2+1}{2\,(x^2-1)^2\sqrt{\frac{-x}{x^2-1}}}.$

45. $\mathcal{D}(f) = \mathbb{R}\setminus\left\{k\dfrac{\pi}{2}, k\in\mathbb{Z}\right\}, \quad f'(x) = 4\operatorname{cotg}(2\,x).$

46. $\mathcal{D}(f) = \mathbb{R}\setminus\{0\}\,, \quad f'(x) = 2\,x\operatorname{arctg}\left(x^{-1}\right) - \dfrac{1}{1+(x^{-1})^2}.$

47. $\mathcal{D}(f) = \langle 0,2), \quad f'(x) = \dfrac{\sqrt{x}(1-2x)}{2+x-x^2} + \dfrac{\ln\left(2+x-x^2\right)}{2\sqrt{x}}.$

48. $\mathcal{D}(f) = (0,1)\,, \quad f'(x) = \dfrac{(1-2x)\cos\left(\ln\left(x-x^2\right)\right)}{x-x^2}\,.$

49. $\mathcal{D}(f) = \langle 0,\infty)\,, \quad f'(x) = \dfrac{4-5x^2}{2\sqrt{x}}\,.$

50. $\mathcal{D}(f) = (-\infty,1\rangle\,, \quad f'(x) = \dfrac{\sin\left(\sqrt{1-x}\right)}{2\sqrt{1-x}}\,.$