2016/2017

Určete definiční obor fce $f(x)$ a spočtěte derivaci $f'(x)$.

1. $f(x) = \ln\left(\dfrac{x^2-3x+2}{x^2-7x+12}\right)$
2. $f(x) = \sqrt{3-\ln(x+1)}$
3. $f(x) = \sqrt{\ln(x^2+x-1)}$
4. $f(x) = \arcsin\left(\dfrac{1}{1-x^2}\right)$
5. $f(x) = \ln\left(\ln\left(\dfrac{x}{x+2}\right)\right)$
6. $f(x) = \operatorname{arctg}\sqrt{3x^2-x-4}$
7. $f(x) = \dfrac{1}{1-\sqrt{x^2-4x+4}}$
8. $f(x) = \sqrt{\ln\left(\dfrac{x-4}{x^2-4}\right)}$
9. $f(x) = \log\Big(\log\big(\log(1-x)\big)\Big)$
10. $f(x) = \ln\Big(\arcsin\big(x^2+3x+3\big)\Big)$
11. $f(x) = \dfrac{1}{\sqrt{\arccos(2x^2+5x+1)}}$
12. $f(x) = \ln\left(\dfrac{x+2}{1-x^2}\right)$
13. $f(x) = \sqrt{\ln(x^2-3x-3)}$
14. $f(x) = \operatorname{arctg}\left(\sqrt{12+x-x^2}\right)$
15. $f(x) = \sqrt{\operatorname{arctg}(x^2+x-6)}$
16. $f(x) = \arccos\left(\dfrac{1-x}{x^2+5x+6}\right)$
17. $f(x) = \ln\left(\dfrac{x^2-1}{x^2-4x+4}\right)$
18. $f(x) = \ln\Big(\ln\big(x^2+3x+3\big)\Big)$
19. $f(x) = \arcsin\left(x+\sqrt{x^2+3x+11}\right)$
20. $f(x) = \sqrt{\ln\left(5-\sqrt{x-5}\right)}$
21. $f(x) = \ln\sqrt{1-\sqrt{x-4}}$
22. $f(x) = \arcsin\sqrt{\dfrac{x-2}{x^2+4}}$
23. $f(x) = \arcsin\left(\dfrac{x-5}{x^2-1}\right)$
24. $f(x) = \ln\Big(\operatorname{arccotg}\big(x^3-x^{-3}\big)\Big)$
25. $f(x) = \ln\left(x+\sqrt{x^2+3x+2}\right)$
26. $f(x) = \arccos(-x^2-x+1)$
27. $f(x) = \sqrt{\arcsin\left(\dfrac{1-x}{3x-1}\right)}$
28. $f(x) = \ln\left(\dfrac{x^2+2x-3}{x^2-2x-3}\right)$
29. $f(x) = \dfrac{1}{1-\sqrt{1-x^2}}$
30. $f(x) = \operatorname{arctg}\left(\dfrac{x^2-4}{16-x^2}\right)$

2016/2017

Určete definiční obor fce $f(x)$ a spočtěte derivaci $f'(x)$.

1. $\mathcal{D}(f) = (-\infty, 1) \cup (2, 3) \cup (4, \infty)$
$$f'(x) = \frac{-2\left(2x^2 - 10x + 11\right)}{\left(x^2 - 3x + 2\right)\left(x^2 - 7x + 12\right)}$$

2. $$\mathcal{D}(f) = \left(-1, -1 + \mathrm{e}^3\right\rangle, \qquad f'(x) = \frac{1}{2\sqrt{3 - \ln(x+1)}} \cdot \frac{-1}{x+1},$$

3. $$\mathcal{D}(f) = (-\infty, -2\rangle \cup \langle 1, \infty) \quad f'(x) = \frac{1}{2\sqrt{\ln\left(x^2 + x - 1\right)}} \cdot \frac{2x+1}{x^2 + x - 1}$$

4. $$\mathcal{D}(f) = \left(-\infty, -\sqrt{2}\right\rangle \cup \left\langle \sqrt{2}, \infty, \right)$$
$$f'(x) = \frac{2x}{\left(1 - x^2\right)^2 \sqrt{1 - \left(\frac{1}{1 - x^2}\right)^2}}$$

5. $$\mathcal{D}(f) = (-\infty, -2) \quad f'(x) = \frac{2}{x(x+2)\ln\left(\frac{x}{x+2}\right)}$$

6. $$\mathcal{D}(f) = (-\infty, -1\rangle \cup \left\langle \frac{4}{3}, \infty\right) \qquad f'(x) = \frac{1}{1 + \left(\sqrt{3x^2 - x - 4}\right)^2} \cdot \frac{1}{2\sqrt{3x^2 - x - 4}} \cdot (6x - 1)$$

7. $$\mathcal{D}(f) = (-\infty, 1) \cup (1, 3) \cup (3, \infty), \qquad f'(x) = \frac{x - 2}{\sqrt{x^2 - 4x + 4}\left(1 - \sqrt{x^2 - 4x + 4}\right)^2}$$

8. $$\mathcal{D}(f) = (-2, 0\rangle \cup \langle 1, 2) \quad f'(x) = \frac{-x^2 + 8x - 4}{2(x-4)\left(x^2 - 4\right)\sqrt{\ln\left(\frac{x-4}{x^2-4}\right)}}$$

9. $$\mathcal{D}(f) = (-\infty, -9), \qquad f'(x) = \frac{1}{\left(\log\left(\log(1-x)\right)\right)\ln 10} \cdot \frac{1}{\left(\log(1-x)\right)\ln 10} \cdot \frac{1}{(1-x)\ln 10},$$

10. $$\mathcal{D}(f) = \langle -2, -1\rangle \quad f'(x) = \frac{2x+3}{\arcsin\left(x^2 + 3x + 3\right)\sqrt{1 - \left(x^2 + 3x + 3\right)^2}}$$

11. $\mathcal{D}(f) = \left\langle -\frac{5}{2}, -2 \right\rangle \cup \left\langle -\frac{1}{2}, 0 \right\rangle \quad f'(x) = \frac{4x+5}{2\left(\arccos\left(2x^2+5x+1\right)\right)^{\frac{3}{2}}\sqrt{1-\left(2x^2+5x+1\right)^2}}$

12. $\mathcal{D}(f) = (-\infty, -2) \cup (-1, 1) \quad f'(x) = \frac{x^2+4x+1}{(x+2)\left(1-x^2\right)}$

13. $\mathcal{D}(f) = (-\infty, -1\rangle \cup \langle 4, \infty) \quad f'(x) = \frac{2x-3}{2\left(x^2-3x-3\right)\sqrt{\ln\left(x^2-3x-3\right)}}$

14. $\mathcal{D}(f) = \langle -3, 4\rangle \quad f'(x) = \frac{1-2x}{2\left(1+\left(\sqrt{12+x-x^2}\right)^2\right)\sqrt{12+x-x^2}}$

15. $\mathcal{D}(f) = (-\infty, -3) \cup (2, \infty) \quad f'(x) = \frac{1}{2\sqrt{\operatorname{arctg}\left(x^2+x-6\right)}} \cdot \frac{2x+1}{1+\left(x^2+x-6\right)^2}$

16. $\mathcal{D}(f) = (-\infty, -5\rangle \cup \langle -1, \infty) \quad f'(x) = \frac{11+2x-x^2}{\left(x^2+5x+6\right)^2\sqrt{1-\left(\frac{1-x}{x^2+5x+6}\right)^2}}$

17. $\mathcal{D}(f) = (-\infty, -1) \cup (1, 2) \cup (2, \infty) \quad f'(x) = \frac{2-4x}{(x-2)\left(x^2-1\right)}$

18. $\mathcal{D}(f) = (-\infty, -2) \cup (-1, \infty), \quad f'(x) = \frac{1}{\ln\left(x^2+3x+3\right)} \cdot \frac{1}{x^2+3x+3} \cdot \left(x^2+3x+3\right),$

19. $\mathcal{D}(f) = \langle -10, -2\rangle, \qquad f'(x) = \frac{1}{\sqrt{1-\left(x+\sqrt{x^2+3x+11}\right)^2}} \cdot \left(1+\frac{2x+3}{2\sqrt{x^2+3x+11}}\right),$

20. $\mathcal{D}(f) = \langle 5, 21\rangle, \quad f'(x) = \frac{1}{\ln\left(5-\sqrt{x-5}\right)} \cdot \frac{1}{5-\sqrt{x-5}} \cdot \frac{-1}{2\sqrt{x-5}}$

21. $\mathcal{D}(f) = \langle 4, 5), \qquad f'(x) = \frac{1}{\sqrt{1-\sqrt{x-4}}} \cdot \frac{1}{2\sqrt{1-\sqrt{x-4}}} \cdot \frac{-1}{2\sqrt{x-4}}$

22.
$$\mathcal{D}(f) = \langle 2, \infty) \quad f'(x) = \frac{1}{\sqrt{1 - \left(\sqrt{\frac{x-2}{x^2+4}}\right)^2}} \cdot \frac{1}{2\sqrt{\frac{x-2}{x^2+4}}} \cdot \frac{-x^2+4x+4}{\left(x^2+4\right)^2}$$

23.
$$\mathcal{D}(f) = (-\infty, -3\rangle \cup \langle 2, \infty) \quad f'(x) = -\frac{x^2-10x+1}{\left(x^2-1\right)^2\sqrt{1 - \left(\frac{x-5}{x^2-1}\right)^2}}$$

24.
$$\mathcal{D}(f) = \mathbb{R}\setminus\{0\} \quad f'(x) = \frac{1}{\operatorname{arccotg}\left(x^3 - x^{-3}\right)} \cdot \frac{-1}{1 + \left(x^3 - x^{-3}\right)^2} \cdot \left(3x^2 - \left(-3x^{-4}\right)\right)$$

25.
$$\mathcal{D}(f) = \left(-\frac{2}{3}, \infty\right), \qquad f'(x) = \frac{1}{x + \sqrt{x^2+3x+2}} \cdot \left(1 + \frac{2x+3}{2\sqrt{x^2+3x+2}}\right),$$

26.
$$\mathcal{D}(f) = \langle -2, -1\rangle \cup \langle 0, 1\rangle \quad f'(x) = \frac{2x+1}{\sqrt{1 - (-x^2-x+1)^2}}$$

27.
$$\mathcal{D}(f) = \left\langle \frac{1}{2}, 1 \right\rangle \quad f'(x) = -\frac{1}{\sqrt{\operatorname{arcsin}\left(\frac{1-x}{3x-1}\right)}} \cdot \frac{1}{\sqrt{1 - \left(\frac{1-x}{3x-1}\right)^2}} \cdot \frac{1}{(3x-1)^2}$$

28.
$$\mathcal{D}(f) = (-\infty, -3) \cup (-1, 1) \cup (3, \infty),$$
$$f'(x) = -\frac{4x^2+12}{\left(x^2+2x-3\right)\left(x^2-2x-3\right)}$$

29.
$$\mathcal{D}(f) = \langle -1, 0) \cup (0, 1\rangle, \qquad f'(x) = \frac{-x}{\sqrt{1-x^2}\left(1 - \sqrt{1-x^2}\right)^2},$$

30.
$$\mathcal{D}(f) = (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$$
$$f'(x) = \frac{24x}{\left(16 - x^2\right)^2 + \left(x^2 - 4\right)^2}$$