Integrujte

$$\int \frac{x}{\sqrt[3]{2+3x^2}}\,\mathrm{d}x. \tag{1}$$

Integrujte

$$\int x^3 \sin x \,\mathrm{d}x. \tag{2}$$

Integrujte

$$\int_0^{\frac{\sqrt{\pi}}{2}} x \operatorname{tg}\left(x^2\right) \mathrm{d}x. \tag{3}$$

Vypočtěte obsah rovinného obrazce ohraničeného křivkami

$$y = (1-x)^3, \quad y = 1 - 3x. \tag{4}$$

Určete a zakreslete definiční obor $f(x,y)$, spočtěte derivaci :

$$f(x,y) = \sqrt{4x - y^2}, \qquad \frac{\partial f}{\partial x}. \tag{5}$$

K funkci $f(x,y)$ určete tečnou rovinu τ a normálu n v bodě $\boldsymbol{A}$.

$$f(x,y) = \sin\left(xy^2 - 1\right), \qquad \boldsymbol{A}\,[1,-1,?]\,. \tag{6}$$

Nalezněte lokální extrémy fce $f(x,y)$.

$$f(x,y) = x^2 - 3y - 3xy + y^3. \tag{7}$$

Vyřešte diferenciální rovnici

$$2y'' + y' = \cos x. \tag{8}$$

............Zkouška z .... Dne.......................... 20...........semestr				Návrh ze cvič.
Příjmení a jméno číslo	Fak.	Roč.	Skup.	

Hodnocení	1	2	3	4	5	6	7	8	9	10	Σ	Pis.	Otáz.	Ústní	Výsl.

① $$\int \frac{x}{\sqrt[3]{2+3x^2}}\,dx = \left| \begin{array}{l} t = \sqrt[3]{2+3x^2} \\ t^3 = 2+3x^2 \\ 3t^2\,dt = 6x\,dx \\ \frac{\cancel{3}t^2\,dt}{\cancel{6}^2 x} = dx \end{array} \right| = \int \frac{\cancel{x}}{t}\cdot\frac{t^2}{2\cancel{x}}\,dt$$

$$\frac{1}{2}\int t\,dt = \frac{1}{4}t^2 = \frac{1}{4}\left(\sqrt[3]{2+3x^2}\right)^2 = \frac{1}{4}\cdot\sqrt[3]{(2+3x^2)^2} + c$$

② $$\int x^3 \sin x\,dx = \left| \begin{array}{ll} \multicolumn{2}{l}{\text{P.P.1.}} \\ u = x^3 & v' = \sin x \\ u' = 3x^2 & v = -\cos x \end{array} \right| =$$

$$-x^3\cos x - \int 3x^2\cdot(-\cos x)\,dx = -x^3\cos x + 3\int x^2\cos x\,dx = \left| \begin{array}{ll} \multicolumn{2}{l}{\text{P.P.2.}} \\ u = x^2 & v' = \cos x \\ u' = 2x & v = \sin x \end{array} \right|$$

$$-x^3\cos x + 3\left(x^2\sin x - \int 2x\sin x\,dx\right) = -x^3\cos x + 3x^2\sin x - 6\int x\sin x\,dx$$

$$\left| \begin{array}{ll} \multicolumn{2}{l}{\text{P.P.3.}} \\ u = x & v' = \sin x \\ u' = 1 & v = -\cos x \end{array} \right| = -x^3\cos x + 3x^2\sin x - 6\left(x\cdot(-\cos x) - \int(-\cos x)\,dx\right)$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\int\cos x\,dx =$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + c =$$

$$= x(6 - x^2)\cos x + (3x^2 - 6)\sin x + c$$

③ $\displaystyle\int_0^{\frac{\sqrt{\pi}}{2}} x\, tg(x^2)\, dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \\ \frac{1}{2x}dt = dx \\ x=0 \to t=0 \\ x=\frac{\sqrt{\pi}}{2} \to t=\frac{\pi}{4} \end{array} \right| = \int \cancel{x} \cdot tg\,t \cdot \frac{1}{2\cancel{x}}\, dt$

$\displaystyle\frac{1}{2}\int_0^{\frac{\pi}{4}} tg\,t\, dt = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sin t}{\cos t}\, dt = \left| \begin{array}{l} u = \cos t \\ du = -\sin t\, dt \\ -\frac{1}{\sin t}du = dt \\ t=0 \to u=\cos 0 = 1 \\ t=\frac{\pi}{4} \to u=\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{array} \right|$

$\displaystyle\frac{1}{2}\int \cancel{\frac{\sin t}{u}} \cdot \cancel{\frac{-1}{\sin t}}\, du = -\frac{1}{2}\int_1^{\frac{\sqrt{2}}{2}} \frac{1}{u}\, du = -\frac{1}{2}\Big[\ln|u|\Big]_1^{\frac{\sqrt{2}}{2}} =$

$\displaystyle -\frac{1}{2}\left(\ln\frac{\sqrt{2}}{2} - \ln 1\right) = -\frac{1}{2}\ln\frac{\sqrt{2}}{2} = -\frac{1}{2}\ln\frac{2^{\frac{1}{2}}}{2} =$

$\displaystyle -\frac{1}{2}\ln 2^{-\frac{1}{2}} = -\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\ln 2 = \underline{\underline{\frac{1}{4}\ln 2}}$

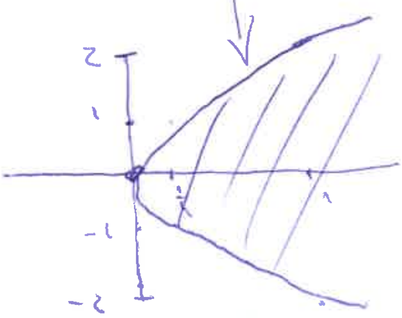
⑤ $f(x,y) = \sqrt{4x - y^2}$

$D(f)$: $4x - y^2 \geq 0$

$4x - y^2 = 0$

Parabola: $x = \frac{1}{4}y^2$

y	0	-1	1	±2
x	0	$\frac{1}{4}$	$\frac{1}{4}$	1



$\displaystyle\frac{\partial f}{\partial x} = \frac{\partial\left((4x-y^2)^{\frac{1}{2}}\right)}{\partial x}$

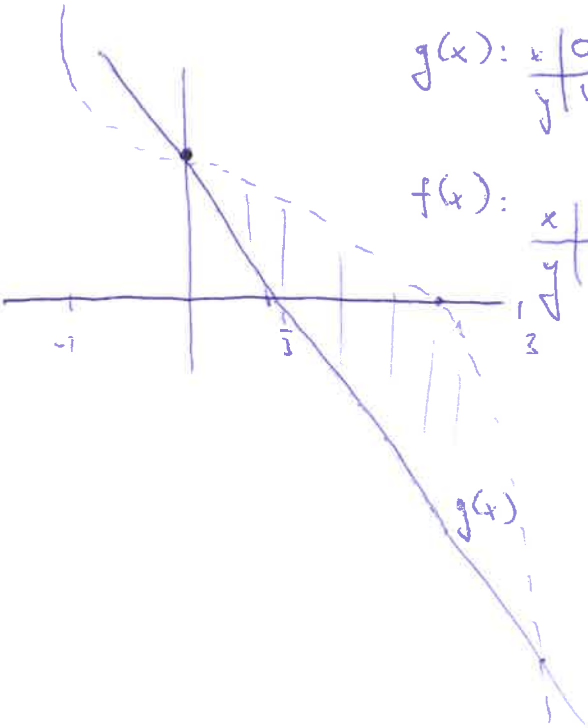
$\displaystyle = \frac{1}{2}(4x-y^2)^{-\frac{1}{2}}\cdot 4 =$

$\displaystyle = \frac{2}{\sqrt{4x-y^2}}$

$\displaystyle\frac{\partial f}{\partial y} = \frac{-2y}{\sqrt{4x-y^2}}$

(4) $y = (1-x)^3 = f(x)$

$y = 1 - 3x = g(x)$



$g(x)$:

x	0	$\frac{1}{3}$
y	1	0

$f(x)$:

x	0	1	-1
y	1	0	$2^3=8$

Průsečíky:

$(1-x)^3 = 1-3x$

$1-3x+3x^2-x^3 = 1-3x$

$3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$\underline{x=0} \wedge \underline{x=3}$

$$\int_0^3 \left((1-x)^3 - (1-3x)\right) dx =$$

$$\int_0^3 (1-3x+3x^2-x^3-1+3x)\, dx =$$

$$\int_0^3 (3x^2-x^3)\, dx = \left[x^3 - \frac{x^4}{4}\right]_0^3 = 27 - \frac{81}{4} - 0 =$$

$$\frac{108-81}{4} = \underline{\underline{\frac{27}{4}}}$$

(6) $f(x,y) = \sin(xy^2-1)$ $\quad A[1,-1,?]$

$f(1,-1) = \sin(1\cdot(-1)^2-1) = \sin 0 = 0$

$$\frac{\partial f}{\partial x} = \cos(xy^2-1)\cdot y^2 \Big|_A = \cos 0 \cdot (-1)^2 = 1$$

$$\frac{\partial f}{\partial y} = \cos(xy^2-1)\cdot 2xy \Big|_A = \cos 0 \cdot 2\cdot 1\cdot(-1) = -2$$

τ: $\frac{\partial f}{\partial x}(A)(x-A_x) + \frac{\partial f}{\partial y}(A)(y-A_y) + f(A) = z$

$1\cdot(x-1) - 2(y+1) + 0 = z$

τ: $x - 2y - z - 3 = 0$

n: $A + t\vec{n}$

$x = 1 + t$

$y = -1 - 2t$

$z = 0 - t, \ t \in \mathbb{R}$

(7) $f(x,y) = x^2 - 3y - 3xy + y^3$

$$\frac{\partial f}{\partial x} = 2x - 3y \qquad 2x - 3y = 0 \rightarrow 2x = 3y \rightarrow x = \frac{3}{2}y$$

$$\frac{\partial f}{\partial y} = -3 - 3x + 3y^2 \qquad -3 - 3x + 3y^2 = 0 \rightarrow -1 - x + y^2 = 0$$

$$-1 - \frac{3}{2}y + y^2 = 0$$

$$\frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} + 4\cdot 1\cdot 1}}{2} = \frac{\frac{3}{2} \pm \sqrt{\frac{25}{4}}}{2} = \frac{\frac{3}{2} \pm \frac{5}{2}}{2}$$

$y = 2 \qquad y = -\frac{1}{2}$

$x = 3 \qquad x = -\frac{3}{4}$

$$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = -3$$

$A\left[-\frac{3}{4}, -\frac{1}{2}\right]$

$B[3, 2]$

$$\frac{\partial^2 f}{\partial y \partial x} = -3 \qquad \frac{\partial^2 f}{\partial y^2} = 6y$$

$A: \begin{vmatrix} 2 & -3 \\ -3 & -3 \end{vmatrix} = -6 - 9 = -15$

NENÍ extrém

$B: \begin{vmatrix} 2 & -3 \\ -3 & 12 \end{vmatrix} = 24 - 9 = 15$

Je extrém

$\frac{\partial^2 f}{\partial x^2}(A) = 2 > 0$

min.

$$\boxed{y = c_1 + c_2 e^{-\frac{1}{2}x} - \frac{2}{5}\cos x + \frac{1}{5}\sin x}$$

(8) $2y'' + y' = \cos x$

$2y'' + y' = 0$

$2\lambda^2 + \lambda = 0$

$\lambda(2\lambda + 1) = 0$

$\lambda = 0 \qquad \lambda = -\frac{1}{2}$

$y = c_1 e^{0x} + c_2 e^{-\frac{1}{2}x}$

$= c_1 + c_2 e^{-\frac{1}{2}x}$

$y_p = A\cos x + B\sin x = ^{(*)} -\frac{2}{5}\cos x + \frac{1}{5}\sin x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

$2(-A\cos x - B\sin x) + (-A\sin x + B\cos x) = \cos x$

$(-2A + B)\cos x + (-A - 2B)\sin x = 0$

$-2A + B = 1 \qquad 5B = 1 \qquad B = \frac{1}{5} \quad (*)$

$-A - 2B = 0 \rightarrow -A = 2B \qquad A = -\frac{2}{5}$